

# Second Batch Community Experiment — Week #1 Problems

First Proof

June 17, 2026

## 1 Problem C1

We consider two online learning frameworks. In regret minimization (RM), the learner has a nonempty compact convex action set and the losses are convex, Lipschitz, and differentiable. In gradient equilibrium (GEQ), the learner's action set is the Euclidean space and the gradients (or vector fields more generally) are bounded and restorative. Prove or disprove: a RM problem can always be solved using a GEQ learner, with no asymptotic loss in the learner's error rate, and vice versa. Formulate reductions using standard assumptions in the online learning literature and state any additional assumptions that are made.

## 2 Problem C2

Let  $\mathcal{C}$  be a category. We will denote by  $c$  the identity map  $\text{id}_c$  of an object  $c$  of  $\mathcal{C}$ .

Let  $k$  be a field. By category, we mean a category enriched in  $k$ -vector spaces.

Given  $\mathcal{L}$  a category, we denote by  $\mathcal{L}^\wedge$  the category of functors from  $\mathcal{L}^{\text{opp}}$  to the category of  $k$ -vector spaces. The fully faithful functor  $\mathcal{L} \rightarrow \mathcal{L}^\wedge$ ,  $l \mapsto \text{Hom}(-, l)$  identifies  $\mathcal{L}$  with a full subcategory of  $\mathcal{L}^\wedge$ .

Let  $\mathcal{C}$  be a monoidal category. The unit object of  $\mathcal{C}$  will be denoted by  $\mathbf{1}$ . We write  $c_1 c_2$  for the tensor product  $c_1 \otimes c_2$  of two objects of  $\mathcal{C}$ . Similarly, we write  $f_1 f_2$  for the tensor product of two maps  $f_1 \otimes f_2$ .

A  $\mathcal{C}$ -module is a strict monoidal functor from  $\mathcal{C}$  to the category of endofunctors of a category. A  $(\mathcal{C}, \mathcal{C})$ -bimodule is a  $(\mathcal{C} \otimes \mathcal{C}^{\text{rev}})$ -module, where  $\mathcal{C}^{\text{rev}}$  is the monoidal category with same underlying category as  $\mathcal{C}$  but with  $c \otimes^{\text{rev}} c' = c' \otimes c$ .

We consider the symmetric monoidal category  $\mathcal{F}$  of vector spaces  $V$  with exhaustive filtrations  $0 = V^{\leq -1} \subset V^{\leq 0} \subset \dots \subset V$ .

Let  $\mathcal{C}$  (resp.  $\mathcal{D}$ ) be a monoidal (resp. symmetric monoidal)  $\mathcal{F}$ -enriched category and  $\Phi : \text{gr } \mathcal{D} \xrightarrow{\sim} \text{gr } \mathcal{C}$  be an equivalence of monoidal categories enriched in graded vector spaces.

Assume there is a full subcategory  $\mathcal{D}_1$  of  $\mathcal{D}^{\leq 0}$  with the following properties:

- (i)  $\mathcal{D}_1$  generates the monoidal category  $\mathcal{D}^{\leq 0}$
- (ii)  $\text{Hom}_{\mathcal{D}}(d_1 \cdots d_n, d'_1 \cdots d'_{n'}) = 0$  for all  $d_1, \dots, d_n, d'_1, \dots, d'_{n'} \in \mathcal{D}_1$  and all  $n \neq n'$
- (iii) for all  $d_1, \dots, d_n, d'_1, \dots, d'_n$  and  $r \geq 0$ , there is an isomorphism

$$\bigoplus_{\substack{w \in \mathfrak{S}_n \\ \ell(w)=r}} \text{Hom}_{\mathcal{D}}^{\leq 0}(d_1 \cdots d_n, d'_{w(1)} \cdots d'_{w(n)}) \xrightarrow{\sim} \left( \text{Hom}_{\mathcal{D}}^{\leq r} / \text{Hom}_{\mathcal{D}}^{\leq r-1} \right)(d_1 \cdots d_n, d'_1 \cdots d'_n)$$

$$(f_w)_w \mapsto \sum_w \sigma_{w^{-1}} \circ f_w$$

where  $\sigma_{w^{-1}}$  is the symmetry isomorphism corresponding to  $w^{-1}$

- (iv) Consider  $d_1, d_2, d_3, d'_1, d'_2, d'_3 \in \mathcal{D}_1$ ,  $\gamma \in \text{Hom}_{\mathcal{D}}^{\leq 0}(d_1 d_2 d_3, d_1 d_3 d_2)$  and  $\gamma' \in \text{Hom}_{\mathcal{D}}^{\leq 0}(d'_1 d'_2 d'_3, d'_1 d'_3 d'_2)$ . If  $u_1 u_3 u_2 \circ \gamma = \gamma' \circ u_1 u_2 u_3$  for all  $u_i \in \text{Hom}_{\mathcal{D}}(d_i, d'_i)$ ,  $i \in \{1, 2, 3\}$ , then  $\gamma = \gamma' = 0$ .

Let  $\delta\mathcal{C}$  be the free  $(\mathcal{C}, \mathcal{C})$ -bimodule generated by one object  $*$  and by maps  $\alpha_c : *c \rightarrow c*$  for  $c \in \mathcal{C}$  modulo the relations

$$\alpha_1 = \text{id}_*$$

$$\alpha_{cc'} = c\alpha_{c'} \circ \alpha_{cc'} : *cc' \rightarrow cc'*$$

$$(f*) \circ \alpha_c = \alpha_{c'} \circ (*f) : *c \rightarrow c' * \text{ for } f \in \text{Hom}_{\mathcal{C}}(c, c').$$

**Problem.** Show that there is a functor  $\Delta$  from  $\mathcal{C}$  to the category of functors of  $(\mathcal{C}, \mathcal{C})$ -bimodules from  $\delta\mathcal{C}$  to  $(\delta\mathcal{C})^\wedge$  such that  $\Delta(c_0)(*)$  is a quotient of  $c_0*$  and  $\Delta(c_0)(\alpha_c) \neq 0$  if  $cc_0 \neq 0$ .